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by
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I. INTRODUCTION

Majority-logic decoding is important for two reasons. First, majority-logic decoding can be very simply implemented, hence, it is attractive from a practical point of view. Next, majority-logic decoding does not inherently perform bounded-distance decoding. Thus it can automatically correct more error patterns than those generated by the decoding algorithm itself without additional cost.

The first use of majority-logic decoding, the decoding algorithm for the repetition code encoded by repeating a message several times, has been lost in antiquity. The first recorded use of majority-logic decoding was the algorithm devised by Reed for the codes discovered by Muller in 1954.[1,2] Reed's ideas were used by some investigators later for their study of majority-logic decoding techniques for cyclic codes.[3-9] In particular, in 1963 Massey presented majority-logic decoding algorithms for block codes that are 1-step and L-step orthogonalizable, where L is the number of majority logic levels required. More importantly, Massey applied majority-logic decoding algorithms to convolutional codes for the first time. In 1960, Gallager developed a slightly different version of majority-logic decoding for his "low-density parity check codes,"[10]

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He showed that Shannon's noise channel theorem could be achieved with this type of decoding algorithm.

Majority-logic decodable codes based on finite geometries were first studied by Rudolph in 1964.[11] Using both Euclidean and projective geometries, Rudolph constructed two large classes of cyclic codes that could be majority-logic decoded. All previously studied majority-logic decodable cyclic codes, including the Reed-Muller codes, are special cases of the finite geometry codes by Rudolph. The problems of determining the generator polynomial, the number of information digits, and the minimum distance of the finite geometry codes were studied by some others.[12-21] In 1968, Goethals and Delsarte, and independently, Weldon used the concept of orthogonal parity check sums to modify the Rudolph's decoding algorithm for the finite geometry codes. The modified decoding algorithm increases the number of error patterns that can be corrected.

In this paper, a survey of recent advances in majority-logic decoding for linear block codes is made. General majority-logic decoding algorithms are reviewed. The decoding schemes for product codes and the finite geometry codes are discussed. Finally, an improved decoding algorithm for the finite geometry codes is described. The improved decoding algorithm greatly reduces the decoding complexity of the codes. It should make the finite geometry codes very attractive in practical use on error-control systems.

II. GENERAL DECODING ALGORITHS

Rudolph's Decoding Algorithm

There are a total of $q^{n-k}-1$ (parity) check sums or vectors that are contained in the null space of an (n,k) code over GF(q). Let S be a matrix formed by a subset of the check sums that have coefficient 1 in the i-th position. In addition, let J be the number of rows in S, and N_j be the number of non-zero elements in the j-th column of S. Then any error digit except the i-th digit can only corrupt N_j of the check sums. The value of the i-th error digit is given by the value assumed by a clear majority of the check sums in S; if no value is assumed by a clear majority, the i-th error digit is assumed to be zero. Using this majority decision rule, the value of the i-th error digit is correctly determined if $[J/2N]$ or fewer errors have occurred, where $[x]$ denotes the integer part of x. This algorithm was proposed by Rudolph to decode high-order finite geometry codes.[22] It takes one-level of majority logic elements to implement the algorithm. If N=1, the check sums are orthogonal. Then the algorithm reduces to the Reed algorithm with L=1, in the next subsection.

The Rudolph's algorithm has been extended by Chow[12] and Weldon.[24] Chow has shown that if the dual code of a code is invariant under a doubly transitive permutation group, then the code can be decoded up to $[n/2(\bar{d}-1]$ errors with one step, where $\bar{d}$ denotes the minimum distance of the dual code. Weldon's work will be discussed in the next section.

Recently, Ng has modified the Rudolph's algorithm in the following way.[23] A set of N all 0's vectors are added to the matrix S so that S consists of J+N rows. Then it can be shown that $[J+N-1/2N]$ or fewer errors can be corrected by majority decision rule. Thus, the modified algorithm can increase the error-correcting ability by one in some cases.

Binary Hamming codes have been shown to be L-step majority-logic decodable.[4,5] Since a binary single-error-correcting code can be obtained from a Hamming code by deleting some of the code digits, a binary 1-error-correcting is L-step majority-logic decodable. Using his modified algorithm, Ng has proved that all binary 1-error-correcting codes are 1-step majority-logic decodable.

The Reed Algorithm

The concept of orthogonal (parity) check sums is the key point of the Reed algorithm.[4] A set of check sums on various error digits is said to be orthogonal on the error digit e_i if e_i is involved in every check sum and no other error digit appears by more than one of the check sums of the set. More generally, a set of check sums is said to be orthogonal on the sum $E = \Sigma c_i e_i$ if E appears in every check sum, and no other error digit is checked by more than one of the check sums in the set, where $C_i \varepsilon GF(q)$. If in a linear code a set of J check sums orthogonal on each digit can be formed, then the code has minimum distance at least equal to J+1.[4]

Suppose that in an (n,k) code at least J check sums orthogonal on each digit can be formed. Then the value of each error digit can be determined by the majority decision rule provided that J/2 or fewer errors occurred. This is 1-step decoding algorithm since one level of majority logic is required. Let $\bar{d}$ denote the minimum distance of the dual code of the code. Then the number of check sums orthogonal on each digit of the code is upper bounded by $n-1/\bar{d}-1$.[24]

If a set of J check sums orthogonal on a linear combination of error digits can be formed, then this sum of error digits can be determined by the majority decision rule. If a number of linear combinations of error digits can be found in this way, then these linear combinations of error digits can be treated as additional check sums. Furthermore, if it is possible to carry out this procedure L-times, each time obtaining J orthogonal check sums, until a set of check sums orthogonal on a single error digit is formed, then the value of the error digit can be corrected provided that J/2 or fewer errors occurred. This decoding procedure is called the Reed algorithm or L-step majority-logic decoding. The number J is upper bounded by $n/\bar{d} - 1/2$ if $\bar{d}$ is even, and $n+1/\bar{d}+1 - 1/2$ if $\bar{d}$ is odd.[24]

Generalized Majority-Logic Decoding Algorithm[25,26]

It is well-known that any d-1 columns of the parity check matrix of a code over GF(q) with minimum distance d are linearly independent. Thus, a set of d-1 vectors in the null space of the code can be constructed in the following way:

$$\begin{array}{cccccccccc} 1 & b_{11} & b_{12} & \cdots & b_{1(n-d)} & a_1 & 0 & \cdots & 0 \\ 1 & b_{21} & b_{22} & \cdots & b_{2(n-d)} & 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 1 & b_{(d-1)1} & b_{(d-1)2} & \cdots & b_{(d-1)(n-d)} & 0 & 0 & \cdots & 0_{(d-1)} \end{array} \qquad (1)$$

where b's and a's are elements of GF(q); and the positions of a's are arbitrary.

A generalized parity check sum on a digit α and (n-d) other digits is defined as a parity check sum which gives the value of the error digit at position α is no errors have occurred in the other (n-d) positions.[25]

From Eq.(1) it is possible to determine a generalized parity check on the first digit and other (n-d) digits by majority decision provided that [d-1/2] or fewer errors occurred. Since the positions of a's are arbitrary, a number of generalized parity check on the first digit and other (n-d) other digits, a generalized check sum on the first digit and any choice of (n-d-1) other digits can be determined. This process can be repeated (n-d) times to yield a set of ordinary check sums orthogonal on the first digit. The other error digits can be determined in a similar way.

The generalized majority-logic decoding algorithm described above can correct at least as many as [d-1/2] errors for any linear block code. However, since the decoding complexity grows exponentially with n-d, the decoding algorithm is impractical, except for the very simple codes, so far as its implementation is concerned.

Threshold Decoding Algorithm

Majority-logic decoding is a special case of threshold decoding.[4] In threshold decoding, the value of an error digit is determined from a set of check sums with a specified threshold value.

In 1966, Townsend and Weldon proposed a modified majority-logic decoding for quasi-cyclic codes.[27] This variable threshold decoding is also applicable to other codes. Basically, variable threshold decoding operates as follows. The decoder attempts to decode each digit of the received word with the decoding threshold set at its highest value d-1 to start with. A change is made on a digit if and only if all d-1 inputs to the majority gate is 1 at the appropriate time. If no change is made for each of the n digits, the threshold is lowered by one. If a change is made, the threshold is immediately increased by one. This process continues until the threshold reaches $[d+1/2]$. Computer simulations have shown that this decoding algorithm is superior to fixed-threshold decoding or ordinary majority-logic decoding.

In 1969, Rudolph has devised a threshold decoding algorithm which can decode any cyclic code up to its minimum distance by a single threshold element.[28] The algorithm is derived from a general decomposition theorem for complex-valued functions. The number of parity check sums required for this algorithm depends on individual code and the solution has not been found in general.

III. APPLICATIONS

In this section, majority-logic decoding algorithms for product codes and finite geometry codes are discussed.

Product Codes

Product codes are attractive for use in many types of data communication systems. The reason is that the decoder of a product code can be implemented as the decoders for its subcodes of shorter length. Let C_1 be an (n_1,k_1) code with minimum distance d_1 and C_2 be an (n_2,k_2) code with minimum distance d_2. Then the product code C of C_1 and C_2 is an (n_1n_2,k_1k_2) code with minimum distance $d = d_1d_2$. The positions of the digits of C can be arranged in an $n_1 x n_2$ rectangular form such that every row forms a code word of C_2 and every column forms a code word of C_1. Because of this property, the decoding of code C can be reduced to the decoding of the subcodes C_1 and C_2.[29] However, some of the patterns of weight $[d_1d_2-1/2]$ or less are not correctable by this decoding scheme, where $[d_1 d_2 - 1/2]$ is the error-correcting capability of the product code.

Recently, Lin and Weldon have shown that the product of two majority-logic decodable codes is also majority-logic decodable provided that one of the subcodes is 1-step decodable.[30] Independently, Gore has obtained more general result.[31] He has been able to show that the product of two majority-logic decodable codes is majority-logic decodable.

An even more general result on the decoding of product codes has been discovered by Reddy.[32] Suppose that C_2 (row code) is 1-step majority-logic decodable. The decoder for the product code first decode C_1 (column) code using any appropriate decoding algorithm. Let e_i be the number of errors corrected in the i-th column. Then $-2e_i$ is assigned as the weight of the error digits in the i-th column. In the second step of the decoding, the column vectors are decoded by the majority-logic decoder for C_2. Assign the weight of a check sum d_1 plus the sum of the weights associated with the digits involved in the check sum if this weight is greater than or equal to zero, otherwise assign 0 as the weight of the check sum. The value of the error digit is given as the value assumed by the check sums whose total weight is larger. Reddy has shown that this decoding algorithm can decode up to minimum distance of the product code provided that one of the subcode is L-step majority-logic decodable.

The Finite Geometry Codes

The finite geometry codes, namely, the Euclidean Geometry (EG) and Projective Geometry (PG) codes, form an important subclass of the class of cyclic codes. Although these codes seem to be somewhat less powerful than the well-known BCH codes, these two types of codes are competitive in many situations. The reason for this is that the finite geometry codes are majority-logic decodable.

The Reed algorithm can be applied to the finite geometry codes. Suppose that all (r+1)-flats of the associated geometry of a finite geometry code are in the null space of the code. Then the Reed algorithm for the code can be described as follows. At the first step, an r-flat is determined from the set of J (r+1)-flats that intersect (or are orthogonal) on the given r-flat. If $[J/2]$ or fewer errors occurred, then the r-flat can be corrected determined by the majority decision on the set of J (r+1)-flats. At the second step, each of the (r-1)-flats is determined from a set of J r-flats that are orthogonal on the given (r-1)-flats. This decoding process continues until the error digits corresponding to all 0-flats are determined. It requires (r+1) steps of majority logic elements for this decoding algorithm.

The decoding complexity for the Reed algorithm grows exponentially with L, the number of levels of majority logic elements required. It is desirable, therefore, to decode the finite geometry codes in as few steps as possible. In this regard, Weldon has proposed two modified decoding algorithms.[24] Both of the modified algorithms can correct as many guaranteed error patterns as the original algorithm. The first of the modified algorithms applies only to certain EG codes and the Reed-Muller codes. Thus, the application of this algorithm is rather limited. In addition, it still requires a large number of decoding steps. The second of the modified algorithms adapts the Rudolph's decoding algorithm. Although this algorithm reduces the number of decoding steps to two, the decoder may not cost less than the decoder using the original algorithm. The reason is that a single majority-logic gate with a very large number of inputs has to be used in the second step of decoding.

An Improved Algorithm

An improved decoding algorithm for the finite geometry code has been discovered recently. Using this improved decoding algorithm, it has been shown that the finite geometry codes can be majority-logic decoded in less than or equal to 3 steps.[33,34] The number of decoding steps depends on the order r of a geometry code and the dimension m of the associated geometry. At the first step of the improved decoding, all r-flats are determined from a given (r+1)-flats in exactly the same way as in the original algorithm. If $r = 1$, then this is the end of the decoding. At the second step, there are two cases to consider. If $r \leq m/2$, then all 0-flats are determined from the r-flats, and the decoding is finished. On the other hand, if $r > m/2$, all $(r-[m/2])$-flats are determined from the r-flats. It requires one more decoding step in this case. At the last step, all 0-flats are determined from the $(r-[m/2])$-flats.

It should be pointed out that while the improved algorithm cuts down the number of decoding steps into less than or equal to 3, it can correct as many guaranteed error patterns as the original Reed algorithm. Therefore, the improved decoding algorithm reduces the decoding complexity of the finite geometry codes enormously in most cases.

IV. CONCLUSIONS

Recent developments in majority-logic decoding for linear block codes has been described. The new decoding algorithms for the finite geometry codes and product codes have changed significantly the outlook of the majority-logic decoding methods. It is expected that they will have great implications in practice of coding the related majority-logic decodable codes.

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